**CHAPTER FOUR**

**CATEGORICAL PROPOSITION**

**Introduction**

This chapter deals categorical propositions. It can be taken as the starting point in the discussion of formal logic. In this chapter we begin with the form of a proposition (statement) that asserts about class (category). Therefore, we will try to see the ***components of categorical proposition, standard forms, and certain attributes of its components (quality, quantity and distribution) and the modern and traditional interpretation of categorical propositions with squares of opposition***. Some activities to be worked by students at the middle and end of the chapter were provided. Answer keys are prepared at the end of the module under the respective activity numbers.

**4.1. The Components of Categorical Propositions**

In Chapter 1 we saw that a proposition (or statement—here we are ignoring the distinction) is a sentence that is either true or false. A proposition that relates two classes, or categories, is called a **categorical proposition.** The classes in question are denoted respectively by the **subject term** and the **predicate term,** and the proposition asserts that either all or part of the class denoted by the subject term is included in or excluded from the class denoted by the predicate term. Here are some examples of categorical propositions:

Example: - *Crocodiles live in water*

* *Majority of Ethiopian athletes are long distance runners*
* *The southern people of Ethiopia are good in football*
* *Most politicians are liars*
* *Banana of Arbaminch is sweaty*
* *Muslims do not believe in gospel of the Bible*
* *Light rays travel at a fixed speed.*

The first statement asserts that the entire class of crocodiles is included in the class of things that live in water. The second statement informs that a certain number of the class of Ethiopian athletes is included in the class of athletes that run long distance. The third statement asserts that the entire class of southern people of Ethiopia is included in the class of people who are good in football. The fort statement asserts that a certain part of the class of politicians is included in the class of population that lies. The fifth statement asserts that the entire class of banana of Arbaminch is included in the class of things that are sweaty. The sixth statement asserts that the entire class of Muslims is excluded from the class of population that believes in the gospel of the bible. The last statement asserts that the entire class of light rays is included in the class of things that travel at a fixed speed.

Since any categorical proposition asserts that either all or part of the class denoted by the subject term is included in or excluded from the class denoted by the predicate term, it follows that there are exactly four types of categorical propositions: (1) those that assert that the whole subject class is included in the predicate class, (2) those that assert that part of the subject class is included in the predicate class, (3) those that assert that the whole subject class is excluded from the predicate class, and (4) those that assert that part of the subject class is excluded from the predicate class. A categorical proposition that expresses these relations with complete clarity is one that is in **standard form.** A categorical proposition is in standard form if and only if it is a substitution instance of one of the following four forms:

All *S* are *P* No *S* are *P*.

Some *S* are *P* Some *S* are not *P*.

Many categorical propositions, of course, are not in standard form because, among other things, they do not begin with the words ‘‘all,’’ ‘‘no,’’ or ‘‘some.’’ In the final section of this chapter we will develop techniques for translating categorical propositions into standard form, but for now we may restrict our attention to those that are already in standard form.

The words ‘‘all,’’ ‘‘no,’’ and ‘‘some’’ are called **quantifiers** because they specify how much of the subject class is included in or excluded from the predicate class. The first form above asserts that the whole subject class is included in the predicate class, the second that the whole subject class is excluded from the predicate class, and so on (Incidentally, in a formal deductive logic the word ‘‘some’’ always means at least one.) The letters ‘‘*S*’’ and ‘‘*P*’’ stand respectively for the subject and predicate terms, and the words ‘‘are’’ and ‘‘are not’’ are called the **copula** because they link (or ‘‘couple’’) the subject term with the predicate term. Consider the following

***. Example***: *All regular local students enrolled in Arbaminch University are students who are sponsored by government*.

This standard-form categorical proposition is analyzed as follows:

*Quantifier:* ***All***  *Copula:* ***Are***

*Subject term: Regular local students enrolled in Arbaminch University*

*Predicate term: students who are sponsored by government*

In resolving standard-form categorical propositions into their four components, one must keep these components separate. They do not overlap each other. In this regard it should be noted that ‘‘subject term’’ and ‘‘predicate term’’ do not mean the same thing in logic that ‘‘subject’’ and ‘‘predicate’’ mean in grammar. The *subject* of the above statement includes the quantifier ‘‘all,’’ but the *subject term* does not. Similarly, the *predicate* includes the copula ‘‘are,’’ but the *predicate term* does not.

Two additional points should be noted about standard-form categorical propositions. The first is that the form ‘‘All *S* are not *P*’’ is *not* a standard form. This form is ambiguous and can be rendered as either ‘‘No *S* are *P*’’ or ‘‘Some *S* are not *P*,’’ depending on the content. The second point is that there are exactly three forms of quantifiers and two forms of copulas. Other texts allow the various forms of the verb ‘‘to be’’ (such as ‘‘is,’’ ‘‘is not,’’ ‘‘will,’’ and ‘‘will not’’) to serve as the copula. For the sake of uniformity, this book restricts the copula to ‘‘are’’ and ‘‘are not.’’ The last section of this chapter describes techniques for translating these alternate forms into the two accepted ones.

The theory of categorical propositions was originated by Aristotle, and it has constituted one of the core topics in logic for over 2,000 years. It remains important even today because many of the statements we make in ordinary discourse are either categorical proposition as they stand or are readily translatable into them. Standard-form categorical propositions represent an ideal of clarity in language, and a familiarity with the relationships that prevail among them provides a backdrop of precision for all kinds of linguistic usage. In Chapter 5 we will see how categorical propositions may be combined to produce *categorical syllogisms,* a kind of argumentation that is closely related to the most basic forms of human reasoning.

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| **🕮 Activity 4.1**  In the following categorical propositions identify the quantifier, subject term, copula, and predicate term.   1. Some lakes are saline water bodies. 2. No universities are financial institutions. 3. All local contractors for main roads are companies financially weak. 4. Some opposition parties of Ethiopia are not parties capable to take power. 5. Some preachers who are intolerant of others’ beliefs are not television evangelists. 6. Some universities that emphasize research are not institutions that neglect undergraduate education. |

**4.2. Quality, Quantity, and Distribution**

Quality and quantity are attributes of categorical propositions. In order to see how these attributes, pertain, it is useful to rephrase the meaning of categorical propositions in class terminology:

**Proposition Meaning in class notation**

All *S* are *P*. Every member of the *S* class is a member of the *P* class;

that is, the *S* class is included in the *P* class.

No *S* are *P*. No member of the *S* class is a member of the *P* class; that is,

the *S* class is excluded from the *P* class.

Some *S* are *P*. At least one member of the *S* class is a member of the *P* class.

Some *S* are not *P*. At least one member of the *S* class is not a member of the *P* class.

The **quality** of a categorical proposition is either affirmative or negative depending on whether it affirms or denies class membership. Accordingly, ‘‘All *S* are *P*’’ and ‘‘Some *S* are *P*’’ have **affirmative** quality, and ‘‘No *S* are *P*’’ and ‘‘Some *S* are not *P*’’ have **negative** quality.

The **quantity** of a categorical proposition is either universal or particular depending on whether the statement makes a claim about *every* member or just *some* member of the class denoted by the subject term. ‘‘All *S* are *P*’’ and ‘‘No *S* are *P*’’ each assert something about every member of the *S* class and thus are **universal.** ‘‘Some *S* are *P*’’ and ‘‘Some *S* are not *P*’’ assert something about one or more members of the *S* class and hence are **particular.**

Note that the quantity of a categorical proposition may be determined through mere inspection of the quantifier. ‘‘All’’ and ‘‘no’’ immediately imply universal quantity, while ‘‘some’’ implies particular. But categorical propositions have no ‘‘qualifier.’’ In universal propositions the quality is determined by the quantifier, and in particular propositions it is determined by the copula.

It should also be noted that particular propositions mean no more and no less than the meaning assigned to them in class notation. The statement ‘‘Some *S* are *P*’’ does *not* imply that some *S* are not *P*, and the statement ‘‘Some *S* are not *P*’’ does *not* imply that some *S* are *P.* It often *happens*, of course, that substitution instances of these statement forms are both true. For example, ‘‘some apples are red’’ is true, as is ‘‘some apples are not red.’’ But the fact that one is true does not *necessitate* that the other be true.

‘‘Some zebras are animals’’ is true (because at least one zebra is an animal), but ‘‘some zebras are not animals’’ is false. Similarly, ‘‘some turkeys are not fish’’ is true, but ‘‘some turkeys are fish’’ is false. Thus, the fact that one of these statement forms is true does not *logically imply* that the other is true, as these substitution instances clearly prove.

Since the early Middle Ages the four kinds of categorical propositions have commonly been designated by letter names corresponding to the first four vowels of the Roman alphabet: **A, E, I, O.** The universal affirmative is called an **A** proposition, the universal negative an **E** proposition, the particular affirmative an **I** proposition, and the particular negative an **O** proposition. The material presented thus far in this section may be summarized as follows:

**Proposition Letter name Quantity Quality**

All *S* are *P*. **A** universal affirmative

No *S* are *P*. **E** universal negative

Some *S* are *P*. **I** particular affirmative

Some *S* are not *P*. **O** particular negative

Unlike quality and quantity, which are attributes of *propositions*, **distribution** is an attribute of the *terms* (subject and predicate) of propositions. A term is said to be distributed if the proposition makes an assertion about every member of the class denoted by the term; otherwise, it is undistributed. Stated another way, a term is distributed if and only if the statement assigns (or distributes) an attribute to every member of the class denoted by the term. Thus, if a statement asserts something about every member of the *S* class, then *S* is distributed; if it asserts something about every member of the *P* class, then *P* is distributed; otherwise *S* and *P* are undistributed.

Let us imagine that the members of the classes denoted by the subject and predicate terms of a categorical proposition are contained respectively in circles marked with the letters ‘‘*S*’’ and ‘‘*P*.’’ The meaning of the statement ‘‘All *S* are *P*’’ may then be represented by the following diagram:

P

The *S* circle is contained in the *P* circle, which represents the fact that every member of *S* is a member of *P*. (Of course, should *S* and *P* represent terms denoting identical classes, the two circles would overlap exactly.) Through reference to the diagram, it is clear that ‘‘All *S* are *P*’’ makes a claim about every member of the *S* class, since the statement says that every member of *S* is in the *P* class. But the statement does not make a claim about every member of the *P* class, since there may be some members of *P* that are outside of *S*. Thus, by the definition of ‘‘distributed term’’ given above, *S* is distributed and *P* is not. In other words, for any universal affirmative (**A**) proposition, the subject term, whatever it may be is distributed, and the predicate term is undistributed.

Let us now consider the universal negative (**E**) proposition. ‘‘No *S* are *P*’’ states that the *S* and *P* classes are separate, which may be represented as follows: S P

This statement makes a claim about every member of *S* and every member of *P.* It asserts that every member of *S* is separate from every member of *P*, and also that every member of *P* is separate from every member of *S*. Accordingly, by the definition above, both the subject and predicate terms of universal negative (**E**) propositions are distributed.

The particular affirmative (**I**) proposition states that at least one member of *S* is a member of *P*. If we represent this one member of *S* that we are certain about by an asterisk, the resulting diagram looks like this: P

Since the asterisk is inside the *P* class, it represents something that is simultaneously an *S* and a *P*; in other words, it represents a member of the *S* class that is also a member of the *P* class. Thus, the statement ‘‘Some *S* are *P*’’ makes a claim about one member (at least) of *S* and also one member (at least) of *P*, but not about all members of either class. Hence, by the definition of distribution, neither *S* nor *P* is distributed.

The particular negative (**O**) proposition asserts that at least one member of *S* is not a member of *P*. If we once again represent this one member of *S* by an asterisk, the resulting diagram is as follows: \*S P

Since the other members of *S* may or may not be outside of *P*, it is clear that the statement ‘‘Some *S* are not *P*’’ does not make a claim about every member of *S*, so *S* is not distributed. But, as may be seen from the diagram, the statement does assert that the entire *P* class is separated from this one member of *S* that is outside; that is, it does make a claim about every member of *P.* Thus, in the particular negative (**O**) proposition, *P* is distributed and *S* is undistributed.

At this point the notion of distribution may be somewhat vague and elusive. Unfortunately, there is no simple and easy way to make the idea graphically clear. The best that can be done is to repeat some of the things that have already been said. First of all, distribution is an attribute or quality that the subject and predicate terms of a categorical proposition may or may not possess, depending on the kind of proposition. If the proposition in question is an **A** type, then the subject term, whatever it may be, is distributed. If it is an **E** type, then both terms are distributed; if an **I** type, then neither; and if an **O** type, then the predicate. If a certain term is *distributed* in a proposition, this simply means that the proposition says something about every member of the class that the term denotes. If a term is *undistributed*, the proposition does not say something about every member of the class.

An easy way to remember the rule for distribution is to keep in mind that universal (**A** and **E**) statements distribute their subject terms and negative (**E** and **O**) statements distribute their predicate terms.

The material of this section may now be summarized as follows:

**Proposition Letter name Quantity Quality Terms distributed**

All *S* are *P*. **A** universal affirmative *S*

No *S* are *P*. **E** universal negative *S* and *P*

Some *S* are *P*.  **I** particular affirmative none

Some *S* are not *P.* **O** particular negative *P*

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| **🕮 Activity 4.2**  Write the letter name, quantity, quality and the terms distributed for the following statements   1. Some African counties are advanced nations 2. All universities are educational institutions 3. Some Ethiopians are not nationalist 4. No animals are rational |

**4.3. Venn Diagrams and the Modern Square of Opposition**

Before discussing about Venn-diagrams let’s make a short look on the concept of modern and tradition interpretations of categorical propositions. Basically the two interpretations are based on two logicians George Boole and Aristotle respectively. The difference between the two is based on conception on existential implication of the four kinds of categorical propositions.

Actually the difference between the two is on universal categorical propositions. In other words both of the interpretations agree on particular statements. Concerning universal categorical propositions, while the traditional (Aristotelian) interpretation says whenever categorical proposition stated it implies that the subject term exists actually in the real world, the modern (Boolean) interpretation says we should not care about the actual existence of the subject term of universal categorical propositions (may or may not exist, i.e. existence is not a concern). But, concerning particular categorical propositions, both agree that whenever a particular categorical proposition stated, the subject term is assumed to actually exist.

Example: *All humans with wing are Ethiopians*

*Some humans with wing are Ethiopians*

From the stand point of traditional interpretation of categorical proposition, the statement “All humans with wing are Ethiopians” means that all members of the class of humans with wing are included in the class of Ethiopians, and it is assumed that humans with wing really exist in the actual world. But, from the stand point of modern interpretation of categorical propositions, the first statement is interpreted as all members of the class of humans with wing are included in the class of Ethiopians, and it is assumed that humans with wing may or may not really exist in the actual world. Since humans with wing do not exist in the actual world, the statement is automatically wrong from traditional interpretation point of view.

But, concerning the second statement, both of the interpretations interpret it as at least one human with wing is included in the class of Ethiopians, and assumes that it actually exist. We will see more on these interpretations later.

**Venn-Diagram**

From the Boolean standpoint, the four kinds of categorical propositions have the following meaning. Notice that the first two (universal) propositions imply nothing about the existence of the things denoted by *S:*

*All S are P. = No members of S are outside P.*

*No S are P. = No members of S are inside P.*

*Some S are P. = At least one S exists, and that S is a P.*

*Some S are not P. = At least one S exists, and that S is not a*

Adopting this interpretation of categorical propositions, the nineteenth-century logician John Venn developed a system of diagrams to represent the information they express. These diagrams have come to be known as Venn diagrams.

A Venn diagram is an arrangement of overlapping circles in which each circle rep-resents the class denoted by a term in a categorical proposition. Because every categorical proposition has exactly two terms, the Venn diagram for a single categorical proposition consists of two overlapping circles. Each circle is labeled so that it represents one of the terms in the proposition. Unless otherwise required, we adopt the convention that the left-hand circle represents the subject term, and the right-hand circle the predicate term. Such a diagram looks like this:

S P

The members of the class denoted by each term should be thought of as situated inside the corresponding circle. Thus, the members of the *S* class (if any such members exist) are situated inside the *S* circle, and the members of the *P* class (if any such members exist) are situated inside the *P* circle. If any members are situated inside the area where the two circles overlap, then such members belong to both the *S* class and the *P* class. Finally, if any members are situated outside both circles, they are members of neither *S* nor *P.*

Suppose, for example, that the *S* class is the class of Americans and the *P* class is the class of farmers. Then, if we use numerals to identify the four possible areas, the diagram looks like this:

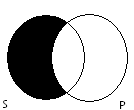
2 3

American 4 Farmers

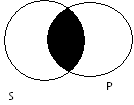
Anything in the area marked "1" is an American but not a farmer, anything in the area marked "2" is both an American and a farmer, and anything in the area marked "3" is a farmer but not an American. The area marked "4" is the area outside both circles; thus, anything in this area is neither a farmer nor an American.

We can now use Venn diagrams to represent the information expressed by the four kinds of categorical proposition. To do this we make a certain kind of mark in a diagram. Two kinds of marks are used: shading an area and placing an X in an area. Shading an area means that the shaded area is empty, and placing an X in an area means that at least one thing exists in that area. The X may be thought of as representing that one thing. If no mark appears in an area, this means that nothing is known about that area; it may contain members or it may be empty. Shading is always used to represent the content of universal (A and E) propositions, and placing an X in an area is always used to represent the content of particular (I and 0) propositions. The content of the four kinds of categorical propositions is represented as follows:

A: All S are P I: Some S are P

 S P

E: No S are P O: Some S are not P

 S P

Recall that the A proposition asserts that no members of *S* are outside *P.* This is represented by shading the part of the *S* circle that lies outside the *P* circle. The E proposition asserts that no members of *S* are inside *P.* This is represented by shading the part of the *S* circle that lies inside the *P* circle. The I proposition asserts that at least one *S* exists and that *S* is also a *P.* This is represented by placing an X in the area where the *S* and *P* circles overlap. This X represents an existing thing that is both an *S* and a *P.* Finally, the **0** proposition asserts that at least one *S* exists, and that *S* is not a *P.* This is represented by placing an X in the part of the *S* circle that lies outside the *P* circle. This X represents an existing thing that is an *S* but not a *P.*

Because there is no X in the diagrams that represent the universal propositions, these diagrams say nothing about existence. For example, the diagram for the A proposition merely asserts that nothing exists in the part of the *S* circle that lies outside the *P* circle. The areas where the two circles overlap and the part of the *P* circle that lies out-side the *S* circle contain no marks at all. This means that something might exist in these areas, or they might be completely empty. Similarly, in the diagram for the **E** proposition, no marks appear in the left-hand part of the *S* circle and the right-hand part of the *P* circle. This means that these two areas might contain something or, on the other hand, they might not.

**The Modern Square of Opposition**

Let us compare the diagram for the **A** proposition with the diagram for the **0** proposition. The diagram for the **A** proposition asserts that the left-hand part of the *S* circle is empty, whereas the diagram for the **0** proposition asserts that this same area is not empty. These two diagrams make assertions that are the exact opposite of each other. As a result, their corresponding statements are said to contradict each other. Analogously, the diagram for the **E** proposition asserts that the area where the two circles overlap is empty, whereas the diagram for the **I** proposition asserts that the area where the two circles overlap is not empty. Accordingly, their corresponding propositions are also said to contradict each other. This relationship of mutually contradictory pairs of propositions is represented in a diagram called the **modern square of opposition.** This diagram, which arises from the modern (or Boolean) interpretation of categorical propositions, is represented as follows: A Logically undetermined E

Contradictory

Logically undetermined

Logically undetermined

Contradictory

I Logically undetermined O

If two propositions are related by the **contradictory relation,** they necessarily have opposite truth value. Thus, if certain **A** proposition is given as true, the corresponding 0 proposition must be false. Similarly, if certain **I** proposition is given as false, the corresponding **E** proposition must be true. But no other inferences are possible. In particular, given the truth value of an **A** or **0** propositions, nothing can be determined about the truth value of the corresponding **E** or I proposition. These propositions are said to have **logically undetermined truth value.** Like all propositions, they do have a truth value, but logic alone cannot determine what it is. Similarly, given the truth value of an **E** or **I** proposition, nothing can be determined about the truth value of the corresponding **A** or **0** propositions. They, too, are said to have logically undetermined truth value.

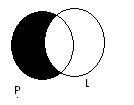
**Testing Immediate Inferences**

Immediate inferences are arguments having one premise and a conclusion, instead of reasoning from one premise to the next, and then to the conclusion, we proceed immediately to the conclusion. Since the modern square of opposition provides logically necessary results, we can use it to test certain arguments for validity. We begin by assuming the premise is true, and we enter the pertinent truth value in the square. We then use the square to compute the truth value of the conclusion. If the square indicates that the conclusion is true, the argument is valid; if not, the argument is invalid. Examples:

1. *Some politicians are liars. Therefore, it is false that, no politicians are liars*
2. *All politicians are liars. Therefore, some politicians are liars*

To test the validity of this argument, we can use the square of opposition to compute it. We start by assuming the premise true. Since the conclusion says the statement no politicians are liars is false that it is true. So by contradictory relation the argument is valid. Arguments that are valid from the Boolean standpoint are said to be **unconditionally valid** because they are valid regardless of whether their terms refer to existing things. Concerning the second argument since Modern Square of opposition does not tell us the relationship between A and I statement, assuming the premise true, the truth value of the conclusion is undetermined. So the argument is invalid.

We can also test the validity of immediate inferences by Venn-Diagram. Let us look the second argument from the above argument. First, let us represent politician by ‘P’ and liars by ‘L’ and see the validity: All politicians are liars Therefore, some politicians are liars

 P L

Since the information contained in the premise is not similar with the conclusion, the argument is invalid. Now let us look the first argument from the above example.

Some politicians are liars Therefore, it is false that, no politicians are liars

P L P L

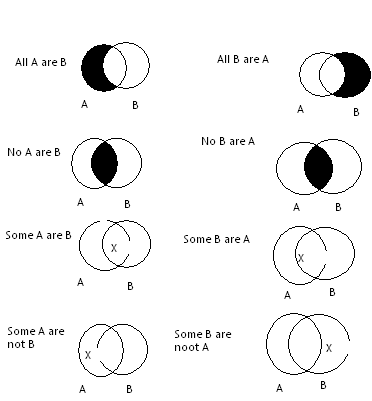
To say ‘some politicians are liars’ and ‘it is false that, no politicians are liars’ is the same. The second statement can be interpreted as ‘at least one politician is a liar’, but not ‘All politicians are liars’. Therefore, the argument is valid.

**4.4 Conversion, Obversion, and Contraposition**

Converse, obverse and contrapositions are techniques used to translate and interpret categorical propositions other way than they are stated for simplicity. It may also use to translate none standard form categorical proposition in to a standard form.

**Conversion**

It is a technique which can be simply formed by interchanging/switching the subject and predicate terms. If ‘A’ is subject term and ‘B’ is predicate term, the following Venn-diagrams show how the converse of all categorical propositions with the original proposition.



If we examine the diagram for the **E** statement, we see that it is identical to that of its converse. Also, the diagram for the **I** statement is identical to that of its converse. This means that the **E** statement and its converse are logically equivalent, and the **I** statement and its converse are logically equivalent. Two statements are said to be **logically equivalent statements** when they necessarily have the same truth value.

On the other hand, the diagram for the A statement is clearly not identical to the diagram for its converse, and the diagram for the O statement is not identical to the diagram for its converse. Also, these pairs of diagrams are not the exact opposite of each other, as is the case with contradictory statements. This means that an A statement and its converse are logically unrelated as to truth value, and an 0 statement and its converse are logically unrelated as to truth value. In other words, converting an A or 0 statements gives a new statement whose truth value is logically undetermined in relation to the given statement. The converse of an A or O statement does have a truth value, of course, but logic alone cannot tell us what it is.

Because conversion yields necessarily determined results for E and I statements, it can be used as the basis for immediate inferences having these types of statements as premises. The following argument forms are valid:

*No A are B. Therefore, no B are A.*

*Some A are B.*

*Therefore, some B are A.*

Since the conclusion of each argument form necessarily has the same truth value as the premise, if the premise is assumed true, it follows necessarily that the conclusion is true. On the other hand, the next two argument forms are invalid. Each commits the fallacy of illicit conversion: *All A are B. Therefore, all B are A.*

*Some A are not B. Therefore, some B are not A.*

Here are two examples of arguments that commit the fallacy of illicit conversion:

1. *All cats are animals. (True)*

*Therefore, all animals are cats. (False)*

*Fallacy: Illicit Conversion*

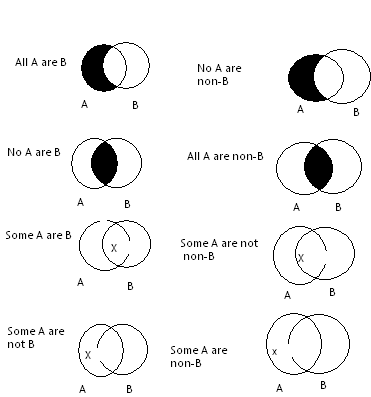
1. *Some animals are not dogs. (True)*

*Therefore, some dogs are not animals. (False)*

*Fallacy: Illicit Conversion*

**Obversion**

**Obversion** requires two steps: (1) changing the quality (without changing the quantity), and **(2)** replacing the predicate with its term complement. The complement of a term is everything outside the class of that term, and formed simply by adding the prefix “none” in front of the term, for example the complement of cat is non-cat (everything outside the class of cat. Let us look the obverse of all kinds of categorical proposition below.

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If we examine these pairs of diagrams, we see that the diagram for each given statement form is identical to the diagram for its obverse. This means that each of the four types of categorical proposition is logically equivalent to (and has the same meaning as) its obverse. Thus, if we obvert an A statement that happens to be true, the resulting statement will be true; if we obvert an **0** statement that happens to be false, the resulting statement will be false, and so on.

As is the case with conversion, obversion can be used to supply the link between the premise and the conclusion of immediate inferences. The following argument forms are valid:

1. *All A are B. Some A are B.*

*Therefore, no A are non-B. Therefore, some A are not non-B.*

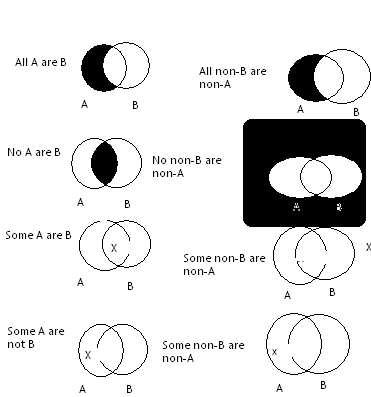
1. *No A are B. Some A are not B.*

*Therefore, all A are non-B. Therefore, some A are non-B.*

Because the conclusion of each argument form necessarily has the same truth value as its premise, if the premise is assumed true, it follows necessarily that the conclusion is true.

**Contraposition**

Like Obversion, contraposition requires two steps: (1) switching the subject and predicate terms and (2) replacing the subject and predicate terms with their term complements. This can be illustrated in the following Venn-diagrams.



Now, inspection of the diagrams for the A and 0 statements reveals that they are identical to the diagrams of their contrapositive. Thus, the A statement and its contra-positive are logically equivalent (and have the same meaning), and the 0 statement and its contrapositive are logically equivalent (and have the same meaning). On the other hand, the diagrams of the E and I statements are neither identical to nor the exact opposite of the diagrams of their contrapositives. This means that contraposing an E or I statement gives a new statement whose truth value is logically undetermined in relation to the given statement.

As with conversion and Obversion, contraposition may provide the link between the premise and the conclusion of an argument. The following argument forms are valid:

1. *All A are B. Therefore, all non-B are non-A.*
2. *Some A are not B.*

*Therefore, some non-B are not non-A.*

On the other hand, the following argument forms are invalid. Each commits the fallacy of illicit contraposition: *Some A are B.*   *Therefore, some non-B are non-A.*

*No A are B. Therefore, no non-B are non-A.*

Here are two examples of arguments that commit the fallacy of illicit contraposition:

*No dogs are cats. (True) Therefore, no non-cats are non-dogs. (False)*

***Fallacy: Illicit Contraposition***

*Some animals are non-cats. (True) Therefore, some cats are non-animals. (False)*

***Fallacy: Illicit Contraposition***

In regard to the first argument, an example of something that is both a non-cat and a non-dog is a pig. Thus, the conclusion implies that no pigs are pigs, which is false. In regard to the second argument, if both premise and conclusion are obverted, the premise becomes "Some animals are not cats," which is true, and the conclusion be-comes "Some cats are not animals," which is false.

Both illicit conversion and illicit contraposition are formal fallacies: They can be detected through mere examination of the form of an argument.

Finally, note that the Boolean interpretation of categorical propositions has prevailed throughout this section. This means that the results obtained are unconditional, and they hold true regardless of whether the terms in the propositions denote actually existing things.

**The Traditional Square of Opposition**

As we have seen above traditional interpretation of, particularly for universal statements, is based on existence of the members of the class denoted by the subject term. Thus, traditional square of opposition shows the relations of all categorical propositions. The relation can be indicated as follows: A Contrary E

T F contradictory

Sub-alternation Sub-alternation

Contradictory T F

I Sub-contrary O

The four relations in the traditional square of opposition may be characterized as follows:

*Contradictory = opposite truth value*

*Contrary = at least one is false (not both true)*

*Sub contrary = at least one is true (not both false)*

*Sub alternation = truth flows downward, falsity flows upward*

The **contradictory relation** is the same as that found in the modern square. Thus, if a certain **A** proposition is given as true, the corresponding 0 proposition is false, and vice versa, and if a certain **A** proposition is given as false, the corresponding **0** proposition is true, and vice versa. The same relation holds between the **E** and I proposition. The contradictory relation thus expresses complete opposition between propositions.

The **contrary relation** differs from the contradictory in that it expresses only partial opposition. Thus, if a certain **A** proposition is given as true, the corresponding **E** proposition is false (because at least one must be false), and if an **E** proposition is given as true, the corresponding A proposition is false. But if an A proposition is given as false, the corresponding **E** proposition could be *either* true or false without violating the "at least one is false" rule. In this case, the E proposition has logically undetermined truth value. Similarly, if an **E** proposition is given as false, the corresponding A proposition has logically undetermined truth value.

These results are borne out in ordinary language. Thus, if we are given the actually true **A** proposition "All cats are animals," the corresponding **E** proposition "No cats are animals" is false, and if we are given the actually true E proposition "No cats are dogs," the corresponding A proposition "All cats are dogs" is false. Thus, the A and **E** propositions cannot both be true. However, they can both be false. "All animals are cats" and "No animals are cats" are both false.

The **sub contrary relation** also expresses a kind of partial opposition. If a certain **I** proposition is given as false, the corresponding **0** proposition is true (because at least one must be true), and if an **0** proposition is given as false, the corresponding **I** proposition is true. But if either an I or an 0 proposition is given as true, then the corresponding proposition could be either true or false without violating the "at least one is true" rule. Thus, in this case the corresponding proposition would have logically un-determined truth value.

Again, these results are borne out in ordinary language. If we are given the actually false **I** proposition "Some cats are dogs," the corresponding **0** proposition "Some cats are not dogs" is true, and if we are given the actually false **0** proposition "Some cats are not animals," the corresponding **I** proposition "Some cats are animals" is true. Thus, the **I** and **0** propositions cannot both be false, but they can both be true. "Some animals are cats" and "Some animals are not cats" are both true.

The **sub alternation relation** is represented by two arrows: a downward arrow marked with the letter T (true), and an upward arrow marked with an F (false). These arrows can be thought of as pipelines through which truth values "flow." The down-ward arrow "transmits" only truth, and the upward arrow only falsity. Thus, if an **A** proposition is given as true, the corresponding **I** proposition is true also, and if an **I** proposition is given as false, the corresponding **A** proposition is false. But if an **A** proposition is given as false, this truth value cannot be transmitted downward, so the corresponding **I** proposition will have logically undetermined truth value. Conversely, if an **I** proposition is given as true, this truth value cannot be transmitted upward, so the corresponding **A** proposition will have logically undetermined truth value. Analogous reasoning prevails for the sub alternation relation between the **E** and **0** propositions. To remember the direction of the arrows for sub alternation, imagine that truth descends from "above," and falsity rises up from "below'

Now that we have examined these four relations individually, let us see how they can be used together to determine the truth values of corresponding propositions. The first rule of thumb that we should keep in mind when using the square to compute more than one truth value is always to use contradiction first. Now, let us sup-pose that we are told that the nonsensical proposition "All adlers are bobkins" is true. Suppose further that adlers actually exist, so we are justified in using the traditional square of opposition. By the contradictory relation, "Some adlers are not bobkins" is false. Then, by either the contrary or the sub-alternation relation, "No adlers are bobkins" is false. Finally, by either-contradictory, sub-alternation, or sub-contrary, "Some adlers are bobkins" is true.

Next, let us see what happens if we assume that "All adlers are bobkins" is false. By the contradictory relation, "Some adlers are not bobkins" is true, but nothing more can be determined. In other words, given a false **A** proposition, both contrary and sub alternation yield undetermined results, and given a true **0** proposition (the one whose truth value we just determined), sub-contrary and sub-alternation yield undetermined results. Thus, the corresponding **E** and **I** proposition have logically undetermined truth value. This result illustrates two more rules of thumb. Assuming that we always use the contradictory relation first, if one of the remaining relations yields a logically undetermined truth value, the others will as well. The other rule is that when-ever one statement turns out to have logically undetermined truth value, its contradictory will also. Thus, statements having logically undetermined truth value will always occur in pairs, at opposite ends of diagonals on the square.

**Immediate Inferences**

Just like the way we used to make immediate inferences based on Modern Square of opposition, the same way is true for Traditional Square of opposition, the difference is while the former tells us the logical relationship between universal affirmative and particular negative, and universal negative and particular affirmative, so that valid inferences of the square is possible only for these relations, but the later makes logical relationship for all statement forms.

One important point is important here. Since traditional interpretation of categorical statements is based on existence, evaluation of actual existence is necessary beyond logical inferences. If inferences made for non-existence entity, then a formal fallacy will occur. In other words, even if the inference is logically valid or based on the correct relation of the square of opposition, it may become invalid when it asserts about non-existence. The names of the fallacy that occur because of invalid inference can be stated by adding a prefix ‘illicit’ before the names of the relationship from which an inference made, for example, if it is from sub-contrary, the fallacy is ‘illicit sub-contrary’, if from sub-alternation, it is called illicit sub-alternation, and so on, if it is because of existence, it is called “existential fallacy’. For Venn-diagram we use a mark of circled “X” for the subject term. This is only for universal statements since there is no difference on particular statements for both interpretations.

Example: *Some fishes are water inhabitants*

*Therefore, all fishes are water inhabitants*

To compute this argument, we start by assuming the premise true and let the square to compute the inference. The inference is made based on sub-alternation (for A and I statements). Since true flows down and false up and the premise is an ‘I’ statement, the conclusion is false by sub-alternation rule. Thus, the argument is invalid. Thus, the argument commits a fallacy of illicit sub-alternation. If we put the conclusion first (as premise) and the premise next (as a conclusion), then we will have a valid inference, and no fallacy will occur. From Aristotelian perspective, the argument can be presented by Venn-diagram as follows.

‘

Some F are W Therefore, All F are W

F W F W

As the diagram indicates though the square of opposition entails invalid inference, Venn-diagram shows that the inference can be valid. And the argument do not commit existential fallacy, fishes actually exist. The following argument is one that commits existential fallacy.

Example: *No fishes that speak human language are water inhabitants*

*Therefore, some fishes that speak human language are not water inhabitants*

Assume the premise true. By sub-alternation relation the inference is a valid inference, but since the argument asserts about non-existent (i.e. fishes that speak human language), it commits existential fallacy; therefore, ultimately the argument becomes invalid.

We can prove this by Venn-diagram as follows:

No F are W therefore, Some F are not W

F W F W

Venn diagram also indicates that the inference is valid. Thus, the inference is valid both by square of opposition and Venn-diagram. But, since we are evaluating the inference from Aristotelian perspective, it commits existential fallacy, and there for ultimately becomes invalid.

|  |
| --- |
| **🕮 Activity 4.4**  Compute the validity of the following arguments through traditional square of opposition   1. Some religious people are fanatic   Therefore, some religious people are not fanatic   1. All walking stones are precious objects   Therefore, no walking stones are precious stones |

**Chapter Summary**

Categorical proposition is a proposition that related two classes. Categorical propositions state that members of a certain class are included in or excluded from other class. The first class is called subject term and the second predicate term. There are four kinds of standard form categorical propositions represented by letter names; A, E, I and O. Quality is the affirmative or negative attribute of a proposition while Quantity is universal or particular attribute of a proposition. Distribution is an attribute of terms. Categorical proposition can be interpreted from modern (Boolean) or traditional (Aristotelian) perspectives. We can also represent information of a categorical proposition by Venn-diagram. Modern square of opposition is a square that shows a contradictory relationship between A and O, and E and I statements, no more other relation, while traditional square opposition is a square that shows relations of all categorical propositions, and it is based on Aristotelian interpretation. Illegitimate immediate inferences from categorical propositions results in formal fallacies.

**👍Self-check box:** Tick honestly on the area that best represent you

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  | Yes | Yes but not clearly | No |
| 1 | Define a categorical proposition |  |  |  |
| 2 | Identify the four kinds of standard form categorical propositions |  |  |  |
| 3 | Construct standard form categorical propositions |  |  |  |
| 4 | Identify the distinguishing features of all standard form categorical propositions |  |  |  |
| 5 | Represent categorical propositions in Venn-diagram |  |  |  |
| 6 | Distinguish between traditional and modern interpretations of categorical propositions |  |  |  |

* In areas where your response entail ‘yes but not clearly’ and ‘no’ go back and revise on that particular area.

**Answer Key**

* **Activity 3.1** 1.Formal 2. Informal 3. Formal 4. Informal
* **Activity 3.2**

1. 1. Appeal to pity 2. Accident 3. Appeal to people (band wagon)

4, Argument against the person (abusive) 5. Missing the point 6. Red herring

1. 1. True 2. True 3. False 4. False

* **Activity 3.3**

1. Hasty generalization 2. Slippery slope 3. Appeal to ignorance 4. False cause (post hoc ergo propter hoc) 5. Weak analogy

* **Activity 3.4**

1. 1. Fallacy not committed 2. Begging the question 3. False dichotomy 4. Equivocation 5. Appeal to ignorance 6. Composition 7. Slippery slope

8. Weak analogy

II) 1. True 2. True 3. False 4. False

* **Activity 4.1**

1. Quantifier – some subject term- lakes copula- are predicate term – saline water bodies
2. Quantifier – No Subject term – universities Copula – are Predicate term – financial institutions
3. Quantifier – All Subject term – local contractors for local roads Copula – are Predicate term – financially weak companies
4. Quantifier – some, Subject term – opposition parties of Ethiopia Copula – are not Predicate term – parties capable to take power
5. Quantifier – Some Subject term – preachers who are intolerant of others’ belief Copula – are not, Predicate term – television evangelists
6. Quantifier – Some Subject term – universities that emphasis research. Copula – are not Predicate term – institutions that neglect undergraduate education

* **Activity 4.2** Q.1 Q.2 Q.3 Q.4

Letter name: I A O E

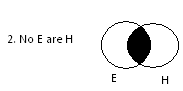
Quantity: particular universal particular universal

Quality: affirmative affirmative negative negative

Term distributed: none subject predicate both

* **Activity 4.3**

1. 1. Some E are C E C



1. Modern square of opposition indicates that the relationship is logically undetermined. Thus, from modern square angle the inference is invalid.

* **Activity 4.4**

1. By sub-alternation inference the argument is valid. It also asserts about actually existing members of the subject term.
2. Though the inference is valid by sub-alternation relation, since it asserts about actually non-existing members of the subject term, the inference is invalid.

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